공학석사 학위논문

IPM Synchronous Machine Speed Controller based on Model Predictive Control Technique

모델 예측 제어 기법에 기반한 매입형 영구자석 동기 전동기의 속도 제어 방법

2012 년 08 월

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전기공학부

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Valeriya Guseva 의 석사학위논문을 인준함

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ABSTRACT

IPM Synchronous Machine Speed Controller based on Model Predictive Control Technique

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This paper deals with application of Model Predictive Control Technique to an Interior Permanent Magnet Synchronous Machine. The speed controller synthesis is done on the base of nonlinear predictive model with respect to nonlinear voltage and current limitations. The controller is implemented as Multiple Input Multiple Output block managing full state-space vector including speed and both currents. Theoretical and practical recommendations on controller design and parameters tuning are given. Several simulation tests are implemented in order to explore the performance of the controller. Furthermore, the qualities of the new controller are being compared to the qualities of the conventional one.

Key words: **IPMSM, Speed Control, Nonlinear Control, Model Predictive Control**
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INTRODUCTION

Model Predictive Control (MPC) is a very topical subject in the advanced drives research field and in the last few years the investigation of this issue has increased significantly. MPC is the only one among the so-called advanced control techniques (usually understood as techniques more advanced than a standard proportional-integral-derivative (PID) control) which has been extremely successful in practical applications in recent decades, exerting a great influence on research and development directions of industrial control systems.

Predictive control presents several advantages that make it suitable for the control of electric drives: it bases on simple and intuitive concepts, it can be adapted to a variety of different control problems and is relatively easy to implement.

An attractive feature of MPC is that it can handle generally constrained nonlinear systems with multiple inputs and outputs (MIMO systems) in a unified and clear manner. The use of all available information of the system to decide the optimal actuation allows to achieve very fast dynamics by avoiding the cascaded structure.

It requires a high amount of real-time calculations, compared to a classical control scheme. However, the development of faster and more powerful microprocessors makes MPC possible to implement.

This work deals with the application of MPC technique to an Interior Permanent Magnet Synchronous Machine (IPMSM) in order to simultaneously control speed and current. Herein, the special attention is given to nonlinearities both of the system as well as constraints. In addition, comparison between MPC controller and typical cascaded PI controller is shown.

I. PHYSICAL STATEMENT OF THE PROBLEM

We shall consider an IPMSM and account for a control target, which is to obtain speed tracking with a given speed reference, under maximum load torque allowed for this speed.

The speed tracking is carried out by obtaining the required voltage reference in order to reach the speed reference value. Based on the measurement data this control signal is then handled by the pulse-width modulation (PWM) inverter which is addresed as an actuator for the machine. A simplified speed control scheme for an IPMSM is shown in Fig. 1.1.

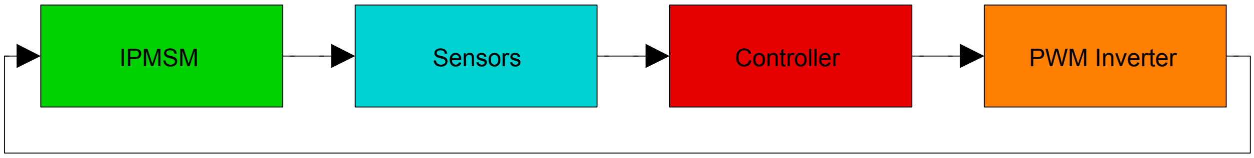

Fig. 1.1. Simplified speed control scheme for an IPMSM.

The inverter, which provides variable voltage to an IPMSM, has limited voltage and current ratings because of the components of the inverter itself and the input voltage of the inverter. Also, even if the inverter has large enough voltage and currents ratings, the machine itself has limitations due to insulation, magnetic saturation and thermal limit.

Hence, the controller must steadily satisfy the following conditions:

- The absolute value of the current amplitude must not exceed the predefined I_s^{max} reference.
- The absolute value of the voltage amplitude must not exceed the predefined V_s^{max} reference.
- The absolute value of speed overshoot must not exceed the predefined value Ω_{rm}^{max} reference.

The inherent dynamics of the PWM inverter is not considered within this work.

II. MATHEMATICAL STATEMENT OF THE PROBLEM

2.1. Mathematical Model of the Plant

The dynamics of an IPMSM can be adequately described from the equations derived for the machine shown in Fig. 2.1 by exchanging the field winding into constant current source, providing the same field flux as the permanent magnet does.

In this chapter, the voltage and electromagnetic torque equations are first established in machine variables. Reference-frame theory set forth in [22] is then used to establish the machine equations with the stator variables in the rotor reference frame. This change of variables allows us to eliminate the time-varying inductances and thereby markedly reduce the complexity of the voltage equations. The resulting differential equations turn out to be nonlinear so computer is being used for further analysis of the electromechanical transient behaviour.

The analysis given in this chapter is valid for a linear magnetic system; saturation is not considered.

Voltage equations in machine variables

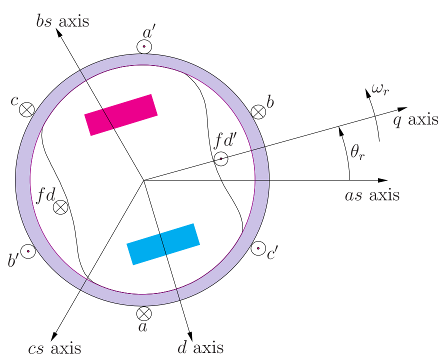

Fig. 2.1. Elementary 2-pole, 3-phase, wye-connected, salient-pole synchronous machine.

It is convenient to begin with the elementary 2-pole, 3-phase, wye-connected salient pole synchronous machine shown in Fig. 2.1 to develop voltage equations. This development may be readily modified to account for a P-pole IPMSM.

Since as, bs, cs windings are considered to be symmetrical, i.e. each winding has the same resistance and the same number of turns, the voltage equations for this machine may be expressed as[1]:

$$\begin{cases} v_{as} = r_s i_{as} + \dfrac{d\lambda_{as}}{dt}, \\ v_{bs} = r_s i_{bs} + \dfrac{d\lambda_{bs}}{dt}, \\ v_{cs} = r_s i_{cs} + \dfrac{d\lambda_{cs}}{dt}, \\ v_{fd} = r_{fd} i_{fd} + \dfrac{d\lambda_{fd}}{dt}, \end{cases} \tag{2.1}$$

where r_s is resistance of the stator winding, r_{fd} is resistance of the field winding and $[v_{as}, i_{as}]$, $[v_{bs}, i_{bs}]$, $[v_{cs}, i_{cs}]$, $[v_{fd}, i_{fd}]$ are voltages and currents of the corresponding windings.
The flux linkages are expressed as:

$$\begin{cases} \lambda_{as} = L_{asas} i_{as} + L_{asbs} i_{bs} + L_{ascs} i_{cs} + L_{asfd} i_{fd}, \\ \lambda_{bs} = L_{bsas} i_{as} + L_{bsbs} i_{bs} + L_{bscs} i_{cs} + L_{bsfd} i_{fd}, \\ \lambda_{cs} = L_{csas} i_{as} + L_{csbs} i_{bs} + L_{cscs} i_{cs} + L_{csfd} i_{fd}, \\ \lambda_{fd} = L_{fdas} i_{as} + L_{fdbs} i_{bs} + L_{fdcs} i_{cs} + L_{fdfd} i_{fd}. \end{cases} \tag{2.2}$$

We can express all the inductances compactly by defining

$$\begin{aligned} L_A &= \pi \mu_0 r l \left(\frac{N_s}{2}\right)^2 \left(\frac{1}{2g_{min}} + \frac{1}{2g_{max}}\right), \\ L_B &= \frac{1}{2} \pi \mu_0 r l \left(\frac{N_s}{2}\right)^2 \left(\frac{1}{2g_{min}} - \frac{1}{2g_{max}}\right), \\ L_{sfd} &= (L_A + L_B) \left(\frac{N_{fd}}{N_s}\right), \\ L_{mfd} &= (L_A + L_B) \left(\frac{N_{fd}}{N_s}\right)^2. \end{aligned} \tag{2.3}$$

In these equations the following quantities are used:

[1]Refer to Chapter 1 and Chapter 5 of [22]

N_s — number of turns of the equivalent phase winding;

N_{fd} — number of turns of the equivalent field winding;

μ_0 — permeability of free space;

r — mean radius of the machine airgap;

l — axial length of the machine airgap;

g_{min} — minimum effective airgap length;

g_{max} — maximum effective airgap length.

The machine inductances may now be written as:

$$
\begin{aligned}
L_{asas} &= L_{ls} + L_A - L_B \cos 2\theta_r,\\
L_{bsbs} &= L_{ls} + L_A - L_B \cos 2\left(\theta_r - \frac{2\pi}{3}\right),\\
L_{cscs} &= L_{ls} + L_A - L_B \cos 2\left(\theta_r + \frac{2\pi}{3}\right),\\
L_{fdfd} &= L_{lfd} + L_{mfd},\\
L_{asbs} &= -\frac{1}{2}L_A - L_B \cos 2\left(\theta_r - \frac{\pi}{3}\right),\\
L_{ascs} &= -\frac{1}{2}L_A - L_B \cos 2\left(\theta_r + \frac{\pi}{3}\right),\\
L_{bscs} &= -\frac{1}{2}L_A - L_B \cos 2(\theta_r + \pi),\\
L_{asfd} &= L_{sfd} \sin\theta_r,\\
L_{bsfd} &= L_{sfd} \sin\left(\theta_r - \frac{2\pi}{3}\right),\\
L_{csfd} &= L_{sfd} \sin\left(\theta_r + \frac{2\pi}{3}\right),
\end{aligned}
\tag{2.4}
$$

where L_{ls}, L_{lfd} are leakage inductances of stator and field windings respectively and θ_r is the electrical angular displacement[2].

It is helpful to incorporate the following substitute variables to the systems (2.1) and (2.2):

$$
\begin{aligned}
i_f &= \frac{2}{3}\left(\frac{N_{fd}}{N_s}\right) i_{fd}; & r_f &= \frac{3}{2}\left(\frac{N_s}{N_{fd}}\right)^2 r_{fd};\\
v_f &= \left(\frac{N_s}{N_{fd}}\right) v_{fd}; & \lambda_f &= \left(\frac{N_s}{N_{fd}}\right) \lambda_{fd}.
\end{aligned}
\tag{2.5}
$$

[2]Which is the same to the actual angular displacement of the rotor for a 2-pole machine

Substituting (2.5) into (2.1) yields the expression for the field voltage:

$$v_f = r_f\, i_f + \frac{d\lambda_f}{dt}. \tag{2.6}$$

Substituting (2.5) into (2.2) and using (2.3), (2.4) yields the expression for the field flux linkage:

$$\begin{aligned}\lambda_f = i_{as}(L_A + L_B)\sin\theta_r + i_{bs}(L_A + L_B)\sin\left(\theta_r - \frac{2\pi}{3}\right) + \\ + i_{cs}(L_A + L_B)\sin\left(\theta_r + \frac{2\pi}{3}\right) + i_f\left(\frac{3}{2}\right)(L_{lfd}\left(\frac{N_s}{N_{fd}}\right)^2 + (L_A + L_B)).\end{aligned} \tag{2.7}$$

Besides, the following three terms of system (2.2) are also modified by (2.5):

$$\begin{aligned} L_{asfd}i_{fd} &= L_{sfd}i_{fd}\sin\theta_r = \frac{3}{2}\,(L_A + L_B)i_f\sin\theta_r, \\ L_{bsfd}i_{fd} &= L_{sfd}i_{fd}\sin\left(\theta_r - \frac{2\pi}{3}\right) = \frac{3}{2}\,(L_A + L_B)i_f\sin\left(\theta_r - \frac{2\pi}{3}\right), \\ L_{csfd}i_{fd} &= L_{sfd}i_{fd}\sin\left(\theta_r + \frac{2\pi}{3}\right) = \frac{3}{2}\,(L_A + L_B)i_f\sin\left(\theta_r - \frac{2\pi}{3}\right). \end{aligned} \tag{2.8}$$

Now it is time to account for an IPMSM where there is no actual field winding and the field flux is produced by the permanent magnet. This leads to elimination of (2.6) and elimination of the first four terms of (2.7), so that:

$$\lambda_f = \frac{3}{2}\,(L_A + L_B)i_f, \tag{2.9}$$

where i_f is addressed as the constant current source.

Considering all the above modifications and (2.8), the voltage and flux linkage equations for the stator windings in terms of machine variables may be expressed in matrix form as:

$$\mathbf{v}_{abcs} = \mathbf{r}_s\mathbf{i}_{abcs} + \frac{d\boldsymbol{\lambda}_{abcs}}{dt}, \tag{2.10}$$

$$\boldsymbol{\lambda}_{abcs} = [\mathbf{L}_s, \mathbf{L}_{sr}]\begin{bmatrix}\mathbf{i}_{abcs} \\ i_f\end{bmatrix}, \tag{2.11}$$

where "$\boldsymbol{s}$" subscript denotes variables associated with the stator.

In (2.10) and (2.11):

$$(\mathbf{i}_{\boldsymbol{abcs}})^{\boldsymbol{T}} = [i_{as}, i_{bs}, i_{cs}],$$
$$(\mathbf{v}_{\boldsymbol{abcs}})^{\boldsymbol{T}} = [v_{as}, v_{bs}, v_{cs}],$$
$$(\boldsymbol{\lambda}_{\boldsymbol{abcs}})^{\boldsymbol{T}} = [\lambda_{as}, \lambda_{bs}, \lambda_{cs}],$$
$$\mathbf{r}_{\boldsymbol{s}} = \mathrm{diag}[r_s, r_s, r_s],$$

$$\mathbf{L}_{\boldsymbol{sr}} = \left[\frac{3}{2}\left(L_A + L_B\right)\sin\theta_r, \quad \frac{3}{2}\left(L_A + L_B\right)\sin\left(\theta_r - \frac{2\pi}{3}\right), \quad \frac{3}{2}\left(L_A + L_B\right)\sin\left(\theta_r + \frac{2\pi}{3}\right)\right]^T,$$

$$\mathbf{L}_{\boldsymbol{s}} = \begin{bmatrix} L_{ls} + L_A - L_B\cos 2\theta_r, & -\frac{1}{2}L_A - L_B\cos 2\left(\theta_r - \frac{\pi}{3}\right), & -\frac{1}{2}L_A - L_B\cos 2\left(\theta_r + \frac{\pi}{3}\right) \\ -\frac{1}{2}L_A - L_B\cos 2\left(\theta_r - \frac{\pi}{3}\right), & L_{ls} + L_A - L_B\cos 2\left(\theta_r - \frac{2\pi}{3}\right), & -\frac{1}{2}L_A - L_B\cos 2(\theta_r + \pi) \\ -\frac{1}{2}L_A - L_B\cos 2\left(\theta_r + \frac{\pi}{3}\right), & -\frac{1}{2}L_A - L_B\cos 2(\theta_r + \pi), & L_{ls} + L_A - L_B\cos 2\left(\theta_r + \frac{2\pi}{3}\right) \end{bmatrix}.$$

Torque equation in machine variables

The energy stored in the coupling field of an IPMSM may be expressed as[3]:

$$W_f = \frac{1}{2}(\mathbf{L}_{\boldsymbol{s}} - L_{ls}\mathbf{E})\mathbf{i}_{\boldsymbol{abcs}} + (\mathbf{i}_{\boldsymbol{abcs}})^{\boldsymbol{T}}\mathbf{L}_{\boldsymbol{sr}} i_f,$$

where **E** goes for identity matrix.

Now it is time to account for a P-pole machine. The following expression is valid for this type of machine:

$$\theta_r = \left(\frac{P}{2}\right)\theta_{rm},$$

where θ_r — electrical angular displacement, θ_{rm} — actual angular displacement of the rotor[4].

Thus, the electromagnetic torque may be evaluated from:

$$T_e = \frac{\partial W_f}{\partial \theta_{rm}} = \left(\frac{P}{2}\right)\frac{\partial W_f}{\partial \theta_r},$$

$$T_e = \left(\frac{P}{2}\right)\left\{\frac{1}{2}(\mathbf{i}_{\boldsymbol{abcs}})^{\boldsymbol{T}}\frac{\partial}{\partial \theta_r}[\mathbf{L}_{\boldsymbol{s}} - L_{ls}\mathbf{E}]\mathbf{i}_{\boldsymbol{abcs}} + (\mathbf{i}_{\boldsymbol{abcs}})^{\boldsymbol{T}}\frac{\partial}{\partial \theta_r}[\mathbf{L}_{\boldsymbol{sr}}]i_f\right\}. \tag{2.12}$$

[3] Refer to Chapter 1 of [22]

[4] Inherently, after differentiating we get $\omega_r = \left(\frac{P}{2}\right)\omega_{rm}$

The torque and rotor speed are related by:

$$J\left(\frac{2}{P}\right)\frac{d\omega_r}{dt} = T_e - B\left(\frac{2}{P}\right)\omega_r - T_L, \tag{2.13}$$

where J is inertia expressed in $[kg \cdot m^2]$, B is friction coefficient expressed in $[N \cdot m \cdot s]$ and T_L is the positive load torque.

Voltage equations in rotor reference-frame variables

The voltage equations of the stator windings can be expressed in the rotor reference frame by direct application of the reference-frame theory[5]. By setting the speed of the arbitrary reference frame equal to the rotor speed ($\omega = \omega_r$) the voltage equations (2.10) may be written as:

$$\mathbf{v}^r_{\boldsymbol{qd0s}} = \mathbf{r}_s \mathbf{i}^r_{\boldsymbol{qd0s}} + \omega_r \boldsymbol{\lambda}^r_{\boldsymbol{qds}} + \frac{d\boldsymbol{\lambda}^r_{\boldsymbol{qd0s}}}{dt}, \tag{2.14}$$

where $(\boldsymbol{\lambda}^r_{\boldsymbol{qds}})^{\boldsymbol{T}} = [\lambda^r_{ds}, -\lambda^r_{qs}, 0]$.

The flux linkage equations (2.11) may be expressed in the rotor reference frame as:

$$\boldsymbol{\lambda}^r_{\boldsymbol{qd0s}} = [\mathbf{K}^r_s \mathbf{L}_s (\mathbf{K}^r_s)^{-1}, \mathbf{K}^r_s \mathbf{L}_{sr}] \begin{bmatrix} \mathbf{i}^r_{\boldsymbol{qd0s}} \\ i_f \end{bmatrix}, \tag{2.15}$$

where the raised index "$\boldsymbol{r}$" used to denote the rotor reference frame[6].

In (2.15):

$$\mathbf{K}^r_s = \frac{2}{3}\begin{bmatrix} \cos\theta_r & \cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos\left(\theta_r + \frac{2\pi}{3}\right) \\ \sin\theta_r & \sin\left(\theta_r - \frac{2\pi}{3}\right) & \sin\left(\theta_r + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix},$$

$$\theta_r = \int_0^t \omega_r(\xi)d\xi + \theta_r(t_0)^7$$

[5] Refer to Chapter 3 of [22]

[6] Note that, the rotor variable i_f is not transformed

[7] In most cases the initial displacement of the reference frame is selected equal to zero

goes for transformation matrix of the 3-phase variables of stationary circuit elements to the reference frame fixed in the rotor.

Using trigonometric relations, it can be shown that:

$$\mathbf{K}_s^r \mathbf{L}_s (\mathbf{K}_s^r)^{-1} = \begin{bmatrix} L_{ls} + L_{mq} & 0 & 0 \\ 0 & L_{ls} + L_{md} & 0 \\ 0 & 0 & L_{ls} \end{bmatrix}, \tag{2.16}$$

$$\mathbf{K}_s^r \mathbf{L}_{sr} = \begin{bmatrix} 0 \\ L_{md} \\ 0 \end{bmatrix}, \quad \text{where} \quad \begin{aligned} L_{mq} &= \frac{3}{2}(L_A - L_B), \\ L_{md} &= \frac{3}{2}(L_A + L_B). \end{aligned} \tag{2.17}$$

Equations (2.14) and (2.15) are often written in the expanded form. Thus, from (2.14):

$$\begin{cases} v_{qs}^r = r_s i_{qs}^r + \omega_r \lambda_{ds}^r + \dfrac{d\lambda_{qs}^r}{dt}, \\ v_{ds}^r = r_s i_{ds}^r - \omega_r \lambda_{qs}^r + \dfrac{d\lambda_{ds}^r}{dt}, \\ v_{0s} = r_s i_{0s} + \dfrac{d\lambda_{0s}}{dt}. \end{cases} \tag{2.18}$$

Substituting (2.16) and (2.17) into (2.15) yields the expression for the flux linkages:

$$\begin{cases} \lambda_{qs}^r = L_{ls} i_{qs}^r + L_{mq} i_{qs}^r, \\ \lambda_{ds}^r = L_{ls} i_{ds}^r + L_{md}(i_{ds}^r + \lambda_f), \\ \lambda_{0s} = L_{ls} i_{0s}{}^{8}. \end{cases} \tag{2.19}$$

Torque equation in rotor reference-frame variables

The expression for the electromagnetic torque in terms of rotor reference-frame variables may be obtained by substituting the equation of transformation into (2.12). Hence:

$$T_e = \left(\frac{P}{2}\right) [(\mathbf{K}_s^r)^{-1} \mathbf{i}_{qd0s}^r]^T \left\{ \frac{1}{2} \frac{\partial}{\partial \theta_r} [\mathbf{L}_s - L_{ls}\mathbf{E}] (\mathbf{K}_s^r)^{-1} \mathbf{i}_{qd0s}^r + \frac{\partial}{\partial \theta_r} [\mathbf{L}_{sr}] i_f \right\}.$$

[8] Since "0s" variables are independent of ω_r and therefore not associated with a reference frame, a raised index is not assigned

After considerable work the above equation reduces to:

$$T_e = \frac{3}{2}\left(\frac{P}{2}\right)(L_{md}(i_{ds}^r + i_f)i_{qs}^r - L_{mq}i_{qs}^r i_{ds}^r). \tag{2.20}$$

Equation (2.20) is equivalent to:

$$T_e = \frac{3}{2}\left(\frac{P}{2}\right)((L_{md} - L_{mq})i_{ds}^r i_{qs}^r + L_{md}i_f i_{qs}^r). \tag{2.21}$$

Balanced 3-phase system and constraints

Usually an IPMSM is operated with the stator windings arranged so that the voltages and currents form a balanced 3-phase set of *abc* sequence as given by (2.22):

$$\begin{aligned} f_{as} &= F_s \sin\theta_{rf}, \\ f_{bs} &= F_s \sin\left(\theta_{rf} - \frac{2\pi}{3}\right), \\ f_{cs} &= F_s \sin\left(\theta_{rf} + \frac{2\pi}{3}\right), \end{aligned} \tag{2.22}$$

where f can represent either voltage or current, F_s and θ_{rf} represent the corresponding amplitude and angular displacement respectively[9].

Substituting (2.22) into the equation of transformation to the rotor reference frame yields:

$$\begin{aligned} f_{qs}^r &= -F_s \sin(\theta_r(t_0) - \theta_{rf}(t_0)), \\ f_{ds}^r &= F_s \cos(\theta_r(t_0) - \theta_{rf}(t_0)), \\ f_{0s} &= 0, \end{aligned} \tag{2.23}$$

Hence, $0s$ variables of (2.18), (2.19) go into zero. Also, if the amplitude F_s is limited to the specific reference value F_s^{max}, using (2.23) we can express it as:

$$(f_{qs}^r)^2 + (f_{ds}^r)^2 \leqslant (F_s^{max})^2. \tag{2.24}$$

[9]Note that, θ_{rf} and θ_r differ only in the initial position $\theta_{rf}(t_0)$ and $\theta_r(t_0)$, since each has the same angular velocity of ω_r

Mathematical model of the plant

The following nonlinear mathematical model, which is derived from (2.9), (2.13), (2.18), (2.19) and (2.21) is going to be used for further analysis:

$$\begin{cases} \dfrac{di_{ds}^r}{dt} = \dfrac{1}{L_d}\left(-r_s i_{ds}^r + \omega_r L_q i_{qs}^r + v_{ds}^r\right), \\ \dfrac{di_{qs}^r}{dt} = \dfrac{1}{L_q}\left(-r_s i_{qs}^r - \omega_r L_d i_{qs}^r - \omega_r \lambda_f + v_{qs}^r\right), \\ \dfrac{d\omega_r}{dt} = \dfrac{P}{2J}\left(\dfrac{3P}{4}(L_d - L_q) i_{ds}^r i_{qs}^r + \dfrac{3P}{4}\lambda_f i_{qs}^r - B\left(\dfrac{2}{P}\right)\omega_r - T_L\right). \end{cases} \tag{2.25}$$

In this model $L_d = L_{ls} + L_{md}$, $L_q = L_{ls} + L_{mq}$ and together with λ_f are normally three parameters identified or provided by the machine manufacturer. In later sections vectors $x = [i_{ds}^r, i_{qs}^r, \omega_r]^T$ and $u = [v_{ds}^r, v_{qs}^r]^T$ are addressed as the state vector and the control vector respectively.

Using (2.24), the limitations mentioned in Chapter I may be expressed as the following nonlinear constraints:

$$\begin{aligned} (i_{ds}^r)^2 + (i_{qs}^r)^2 \leqslant (I_s^{max})^2, \\ (v_{ds}^r)^2 + (v_{qs}^r)^2 \leqslant (V_s^{max})^2. \end{aligned} \tag{2.26}$$

2.2. Control Problem Statement

In this section the mathematical problem of feedback based controller synthesis, which is to guarantee reference speed tracking under the given physical limitations, is set.

For the given IPMSM, which mathematical model (2.25) is derived in Section 2.1, step speed reference and the corresponding maximum load torque are applied. The initial state vector is selected as $x(t_0) = x_0$.

In addition, speed tracking must be done with respect to nonlinear constraints (2.26) for all $t \in [t_0, +\infty)$ and the absolute value of speed overshoot, which must not exceed some predefined value of Ω_r^{max}.

The solution of the problem is obtained in the following form:

$$u = L(t, x, \omega_r^*), \tag{2.27}$$

where $L \in \Omega_L$, Ω_L is the set of operators, which would fit the control target and conditions announced, ω_r^* — speed reference value.

It is clear that there is not a single operator L to satisfy (2.27). In fact, there is an infinite number, so it is worthwhile to look for some optimal one.

For all controllable motions of system (2.25) let us define the functional:

$$J = J(x, u).$$

Since $x = x(t_0, x_0, u)$ and u is defined as (2.27), the functional can be expressed as:

$$J = J(t_0, x_0, L). \tag{2.28}$$

The problem of selection the optimal control vector leads to minimization of the functional (2.28):

$$J(t_0, x_0, L) \to \inf_{L \in \Omega_L}.$$

Hence, the optimal control vector may be written as:

$$u^* = L^*(t, x, \omega_r^*),$$

where $L^* = \arg \inf_{L \in \Omega_L} J(t_0, x_0, L)$.

2.3. Basic Concepts of MPC Technique

Let us assume that mathematical model of some object under control is represented by system of nonlinear differential equations:

$$\dot{x}(t) = f(t, x(t), u(t)), \quad x(t_0) = x_0, \tag{2.29}$$

where $x \in \mathbb{R}^n$ — state vector, $u \in \mathbb{R}^m$ — control vector.

Also suppose the following conditions to be fulfilled:

$$x(t) \in X, \quad u(t) \in U, \quad \forall t \in [t_0, \infty). \tag{2.30}$$

Here $X \subseteq \mathbb{E}^n$ — a set of feasible state values and $U \subseteq \mathbb{E}^m$ — a set of feasible control values. We also assume $f(t, x, u(t))$ to match the requirements of Cauchy-Lipschitz theorem for any piecewise-continuous $u(t)$, which will guarantee the existence of unique solution of Cauchy problem for system (2.29). In addition we assume that $f(t, 0, 0) \equiv 0$, i.e. system (2.29) has zero equilibrium point.

Suppose the control target to be meeting the following conditions:

$$\lim_{t\to\infty} \|x(t) - r_x(t)\| = 0, \quad \lim_{t\to\infty} \|u(t) - r_u(t)\| = 0, \tag{2.31}$$

where predefined functions $r_x(t) \in \mathbb{E}^n$ and $r_u(t) \in \mathbb{E}^m$ determine some object motion to be achieved.

For all controllable motions of system (2.29) let us define a functional:

$$\tilde{J} = \tilde{J}(x(t), u(t)). \tag{2.32}$$

For instance it can be linear-quadratic functional:

$$\tilde{J}(u(\cdot)) = \int_{t_0}^{\infty} ((x - r_x)^T R(x - r_x) + (u - r_u)^T Q(u - r_u))d\tau.$$

Let us set up the problem of optimal feedback control synthesis, which is to ensure meeting control target (2.31) by minimizing functional (2.32) and taking limitations (2.30) into account.

Together with mathematical model (2.29) we consider the following system of differential equations:

$$\dot{\bar{x}}(\tau) = \bar{f}(\tau, \bar{x}(\tau), \bar{u}(\tau)), \quad \bar{x}(\tau)|_{\tau=t} = x(t), \tag{2.33}$$

where $\bar{x} \in \mathbb{E}^n$ — state vector, $\bar{u} \in \mathbb{E}^m$ — control vector. Also assume that $\bar{x}(\tau) \in X$, $\bar{u}(\tau) \in U$, $\tau \in [t, \infty)$ and that function $\bar{f}$ has the same qualities as f does.

We suppose $\bar{f}$ to have such a qualities that for every feasible control vector $\bar{u}(\tau) \equiv u(\tau)$, the corresponding state vectors of systems (2.29) and (2.33) are enough close to each other with respect to Euclidean norm for every $\tau \in [t, \infty)$. In this connection, system (2.33) is called predictive model towards mathematical model (2.29).

Note that, the process of creating mathematical models implies elimination of nonlinearities, parameter variation, external disturbances, etc., which deflect their motion from the objective one. We suppose the model (2.29) to be accurate enough to take all the above factors into account, which is to say that it can change in operation, but these changes are a priori unknown. However, there is a fixed model (2.33) available, which is initialized by current state vector at the moment $\tau = t$, and helps to predict behaviour of the object under control. Prediction is made by searching for a particular solution of system (2.33) with a given control vector.

Let us set up the control vector $\bar{u} = \bar{u}(\tau)$ for $t \in [t, t + T_p]$ and solve system (2.33) starting from $\bar{x}(\tau)|_{\tau=t} = x(t)$ for the same time interval. The obtained particular solution $\bar{x} = \bar{x}(\tau, x(t), \bar{u}(\cdot))$ is called prediction of control object behaviour with prediction horizon T_p.

It is obvious that due to the difference between the real control object and its predictive model, there will be some margin between real and predicted behaviour.

We can now set up the optimal control problem based on prediction. Suppose the control target to be bringing the system (2.33) to some state, which is determined by the above mentioned vector functions $r_x(t) \in \mathbb{E}^n$, $r_u(t) \in \mathbb{E}^m$.

The quality of predictive model control process is characterized by the following functional:

$$J(t, x(t), \bar{u}(\cdot), T_p, T_c) = \int\limits_{t}^{t+T_p} ((\bar{x} - r_x)^T Q(\bar{x} - r_x) + (\bar{u} - r_u)^T R(\bar{u} - r_u))d\tau, \qquad (2.34)$$

where Q and R are symmetric positive definite matrixes and $T_c \leqslant T_p$ is denoted as the control horizon i.e. the moment of time when:

$$\bar{u}(\tau) = \bar{u}(t + T_c), \quad \tau \in [t + T_c, t + T_p], \quad \bar{u}(\cdot) \in U.$$

Note that, functional (2.34) represents the original functional (2.32) defined on the finite time interval $[t, t + T_p]$.

The problem of selection the optimal control for predictive model leads to minimization of the functional (2.34):

$$J(t, x(t), \bar{u}(\cdot), T_p, T_c) \to \inf_{\bar{u} \in \Omega_{\bar{u}}} \qquad (2.35)$$

on the following set of feasible control vectors:

$$\Omega_{\bar{u}} = \Big\{ \bar{u}(\tau) \in U,\, \tau \in [t, t+T_p] : \bar{u}(\tilde{\tau}) = \bar{u}(t+T_c),\, \tilde{\tau} \in [t+T_c, t+T_p],\, \bar{x}(\tau, x(t), \bar{u}(\tau)) \in X \Big\}.$$

The problem solution:

$$\bar{u}^*(\tau) = \bar{u}^*(\tau, x(t), T_p, T_c) = \arg \inf_{\bar{u}(\cdot) \in \Omega_{\bar{u}}} J(t, x(t), \bar{u}(\cdot), T_p, T_c)$$

specifies optimal towards (2.34) functional program control for predictive model (2.33).

It worth mentioning that due to the difference of real object and predictive model dynamics this solution ensures $\bar{x}(\tau, x(t), \bar{u}^*(\cdot)) \in X, \tau \in [t, t+T_p]$, but can not guarantee $x(\tau, x(t), \bar{u}^*(\cdot)) \in X, \tau \in [t, t+T_p]$.

The main idea of MPC technique suggests application of the above optimal program control $\bar{u}^*(\tau)$ not on the whole prediction horizon, but on it's small part. Thus, the object receives the following control vector [24]:

$$\bar{u}^*(\tau) = \bar{u}^*(\tau, x(t), T_p, T_c), \quad \tau \in [t, t+\Delta t], \tag{2.36}$$

where $\Delta t \ll T_p$.

After implementing (2.36), at the moment $\tau = t + \Delta t$, another prediction is done for the horizon T_p and the initial data $x(t+\Delta t)$, together with solving optimization problem for the time interval $[t+\Delta t, t+\Delta t+T_p]$. The result of optimization is applied during $[t+\Delta t, t+2\Delta t]$ time period and then the whole process is repeated.

Thus, MPC is a discrete feedback based control technique, which is performed at every instant Δt according to the following scheme:

1. Sensing or estimating the state vector $x(t)$.
2. Solving the optimization problem towards (2.34) functional using prediction model (2.33) for $\bar{x}(\tau)|_{\tau=t} = x(t)$ initial conditions.
3. Application of the obtained optimal control vector $\bar{u}^*(\tau, x(t), T_p, T_c)$ during $\tau \in [t, t+\Delta t]$ time period.

It must be pointed out, that both linear and nonlinear systems of differential equations can be used as predictive models.

III. IPMSM SPEED CONTROL BASED ON MPC TECHNIQUE

In case of IPMSM, both mathematical model of the object under control (2.29)[1] as well as predictive model (2.33) are defined by (2.25). Hence, the state vector and the control vector of predictive model are expressed as:

$$\bar{x} = [i^r_{ds}, i^r_{qs}, \omega_r]^T \in X, \qquad X \in \mathbb{E}^n (n = 3),$$
$$\bar{u} = [v^r_{ds}, v^r_{qs}]^T \in U, \qquad U \in \mathbb{E}^m (m = 2),$$

where X and U are sets of feasible state values and control values respectively, represented as:

$$X = \{\bar{x} = [i^r_{ds}, i^r_{qs}, \omega_r]^T : (i^r_{ds})^2 + (i^r_{qs})^2 \leqslant (I^{max}_s)^2,\ |\omega_r| \leqslant |\omega^*_r| + \Omega^{max}_r\},$$
$$U = \{\bar{u} = [v^r_{ds}, v^r_{qs}]^T : (v^r_{ds})^2 + (v^r_{qs})^2 \leqslant (V^{max}_s)^2\}.$$

In the problem of speed control, the particular control vector u (two voltages) is addressed as the input to the machine, while the measured state vector x (two currents and speed) goes for the output. The detailed speed control scheme for an IPMSM is shown in Fig. 3.1.

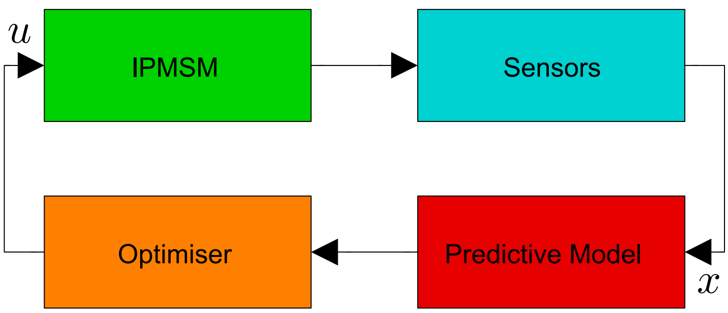

Fig. 3.1. Detailed speed control scheme for an IPMSM.

The quality of predictive model control process is characterized by the following functional:

$$J(t, x(t), \bar{u}(\cdot), T_p, T_c) = \int_t^{t+T_p} (\omega_r - \omega^*_r)^2 d\tau. \tag{3.1}$$

[1]This model is used only in order to simulate the dynamics of the machine, while in reality it is omitted

Thus, the problem of speed control leads to selection of the optimal control vector, which gives minimum to the functional (3.1), i.e

$$\bar{u}^*(\tau) = \bar{u}^*(\tau, x(t), T_p, T_c) = \arg \inf_{\bar{u}(\cdot) \in \Omega_{\bar{u}}} J(t, x(t), \bar{u}(\cdot), T_p, T_c)$$

on the following set of feasible control vectors:

$$\Omega_{\bar{u}} = \left\{ \bar{u}(\tau) \in U,\, \tau \in [t, t+T_p] : \bar{u}(\tilde{\tau}) = \bar{u}(t+T_c),\, \tilde{\tau} \in [t+T_c, t+T_p],\, \bar{x}(\tau, x(t), \bar{u}(\tau)) \in X \right\}.$$

This optimization problem refers to General Problem of Nonlinear Programming and can be solved numerically using Interior-point method [8].

The control process is implemented according to the scheme given in Section 2.3.

IV. NUMERICAL SIMULATIONS

The simulations are performed for the IPMSM, whose parameters are presented in Table 4.1.

Rated speed	$\omega_{rm,rated} = 1750 \left[\frac{rev}{min}\right]$
Maximum load torque	$T_L^{max} = 64.58\,[N \cdot m]$
Rated power	$P_{m,rated} = 11\,[kW]$
Rated line-to-line rms voltage	$V_{rated} = 190\,[V]$
Rated line-to-line rms current	$I_{rated} = 39.5\,[A]$
Number of poles	$P = 6$
Phase resistance	$r_s = 0.15\,[\Omega]$
d-axis inductance	$L_d = 3.6\,[mH]$
q-axis inductance	$L_q = 4.3\,[mH]$
Permanent magnet field flux	$\lambda_f = 0.254\,[Wb]$
Total inertia	$J = 0.01\,[kg \cdot m^2]$
Friction coefficient	$B = 0.018\,[N \cdot m \cdot s]$

Table 4.1. IPMSM parameters

The absolute value of voltage amplitude V_s^{max} is decided by DC link voltage V_{dc} of the PWM inverter, while the absolute value of current amplitude is decided by the IPMSM itself. All the limitations are given in Table 4.2.

Inverter DC link voltage	$V_{dc} = 150\,[V]$
Maximum phase voltage amplitude	$V_s^{max} = \frac{V_{dc}}{\sqrt{3}}\,[V]$
Maximum phase current amplitude	$I_s^{max} = I_{rated}\sqrt{2}\,[A]$
Absolute value of speed overshoot	$\Omega_{rm}^{max} = 0.005 \cdot \lvert\omega_{rm}^*\rvert \left[\frac{rev}{min}\right]$

Table 4.2. IPMSM limitations

The following Table 4.3 represents controller parameters used for simulations.

Prediction horizon	$T_p = 0.1\,[s]$
Control horizon	$T_c = 0.0001\,[s]$
Control period	$\Delta t = 0.0001\,[s]$

Table 4.3. IPMSM controller parameters

The simulation results are shown in the following figures. Fig. 4.1 and Fig. 4.3 represent the output of MPC controller for different speed and load torque references. The results of comparison between MPC and PI controllers are represented by Fig. 4.4 – Fig. 4.6.

All tests are made with respect to zero initial conditions $x(t_0)\Big|_{t_0=0} = 0$, except for the second performance test, which $x(t_0)\Big|_{t_0=0} = \left[-6.29\,[A], 28.84\,[A], 314.16\,\left[\frac{rad}{s}\right]\right]^T$.

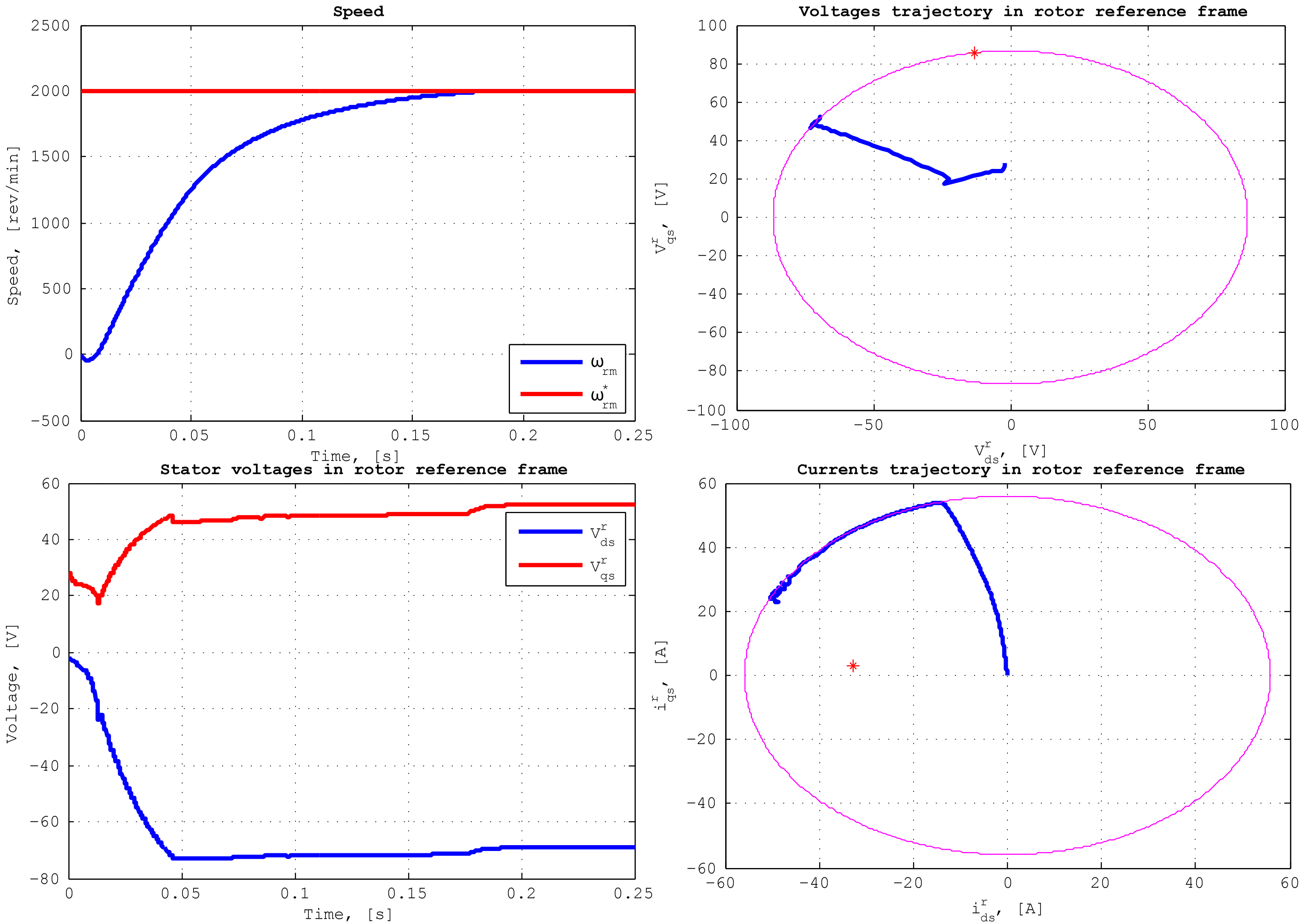

Fig. 4.1. Performance test #1: $\omega^*_{rm} = 2000\,[rev/min]$, $\quad T_L = 0.4\,T_L^{max}\,[N \cdot m]$

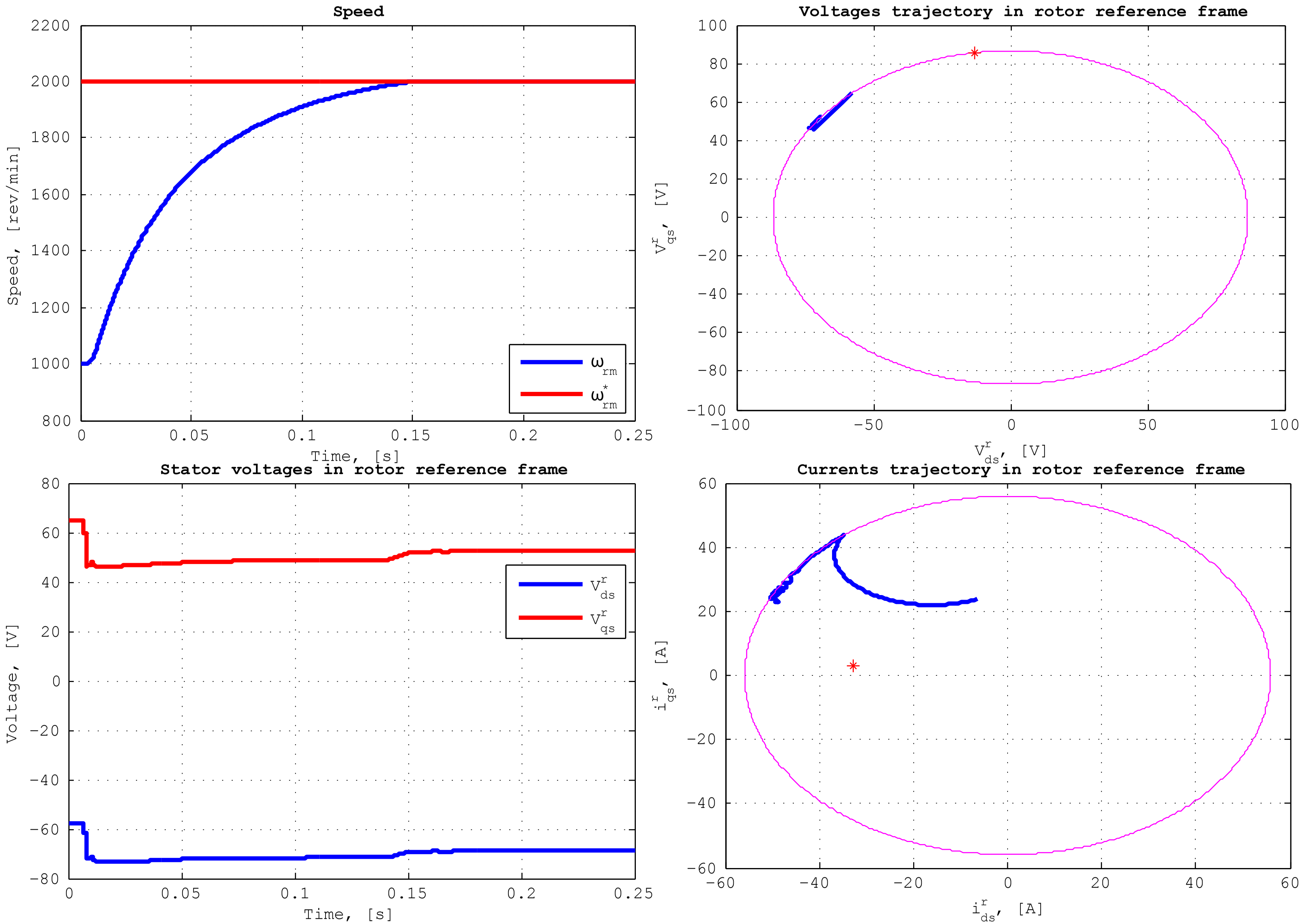

Fig. 4.2. Performance test #2: $\omega^*_{rm} = 2000\,[rev/min]$, $\quad T_L = 0.4\,T_L^{max}\,[N \cdot m]$

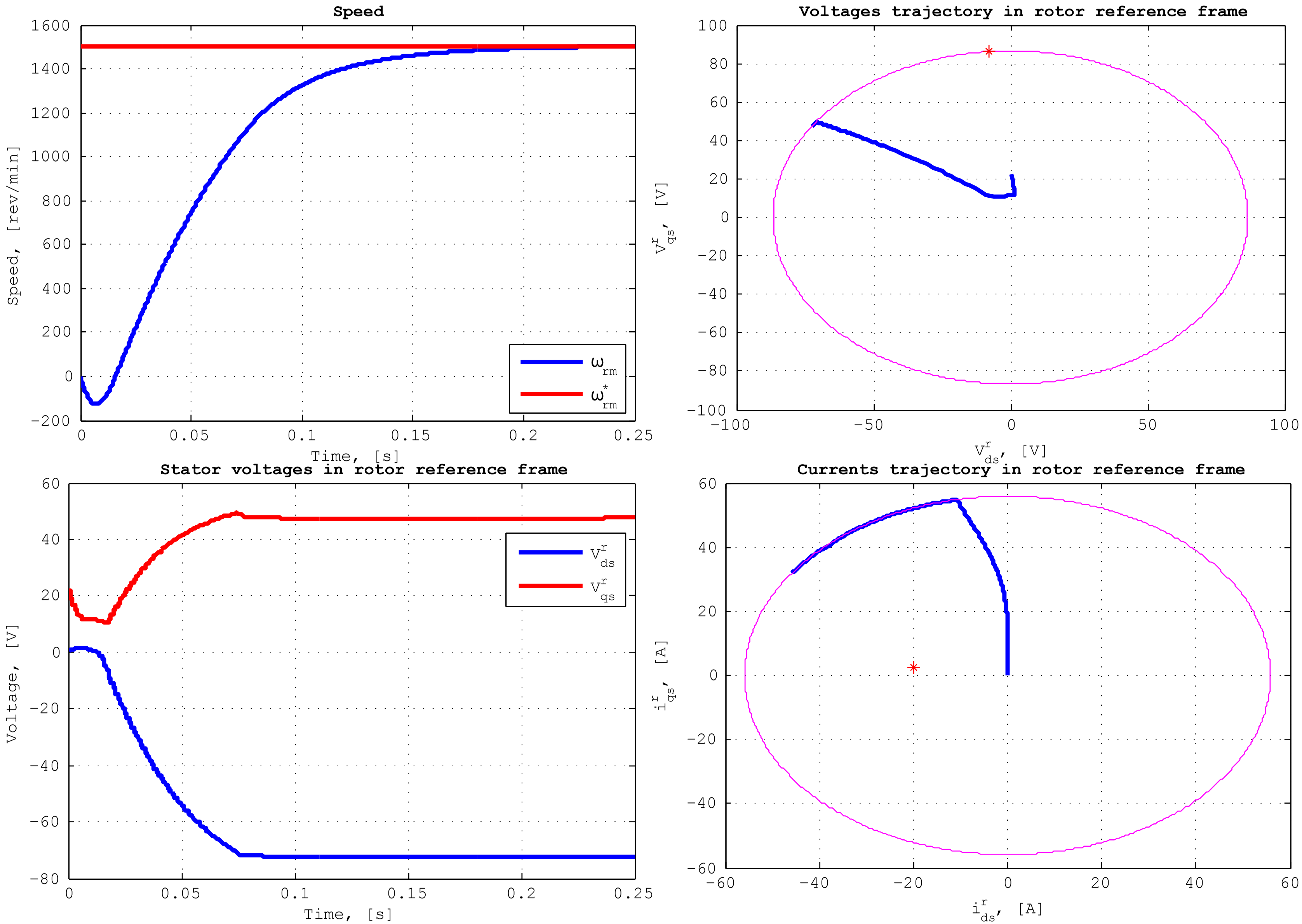

Fig. 4.3. Performance test #3: $\omega^*_{rm} = 1500\,[rev/min]$, $\quad T_L = 0.6\,T_L^{max}\,[N \cdot m]$

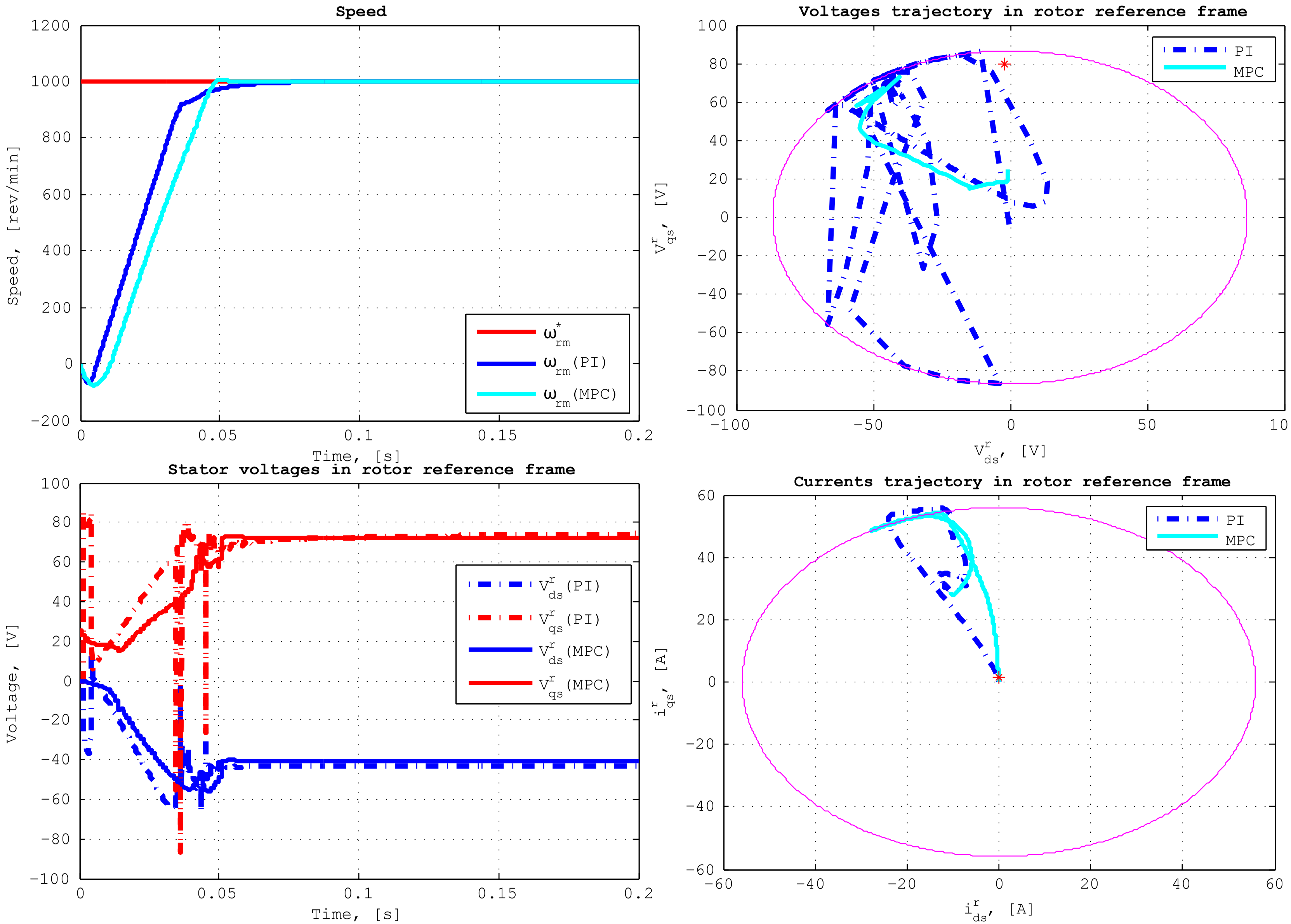

Fig. 4.4. Comparison test #1: $\omega^*_{rm} = 1000\,[rev/min]$, $\quad T_L = 0.5\,T_L^{max}\,[N \cdot m]$

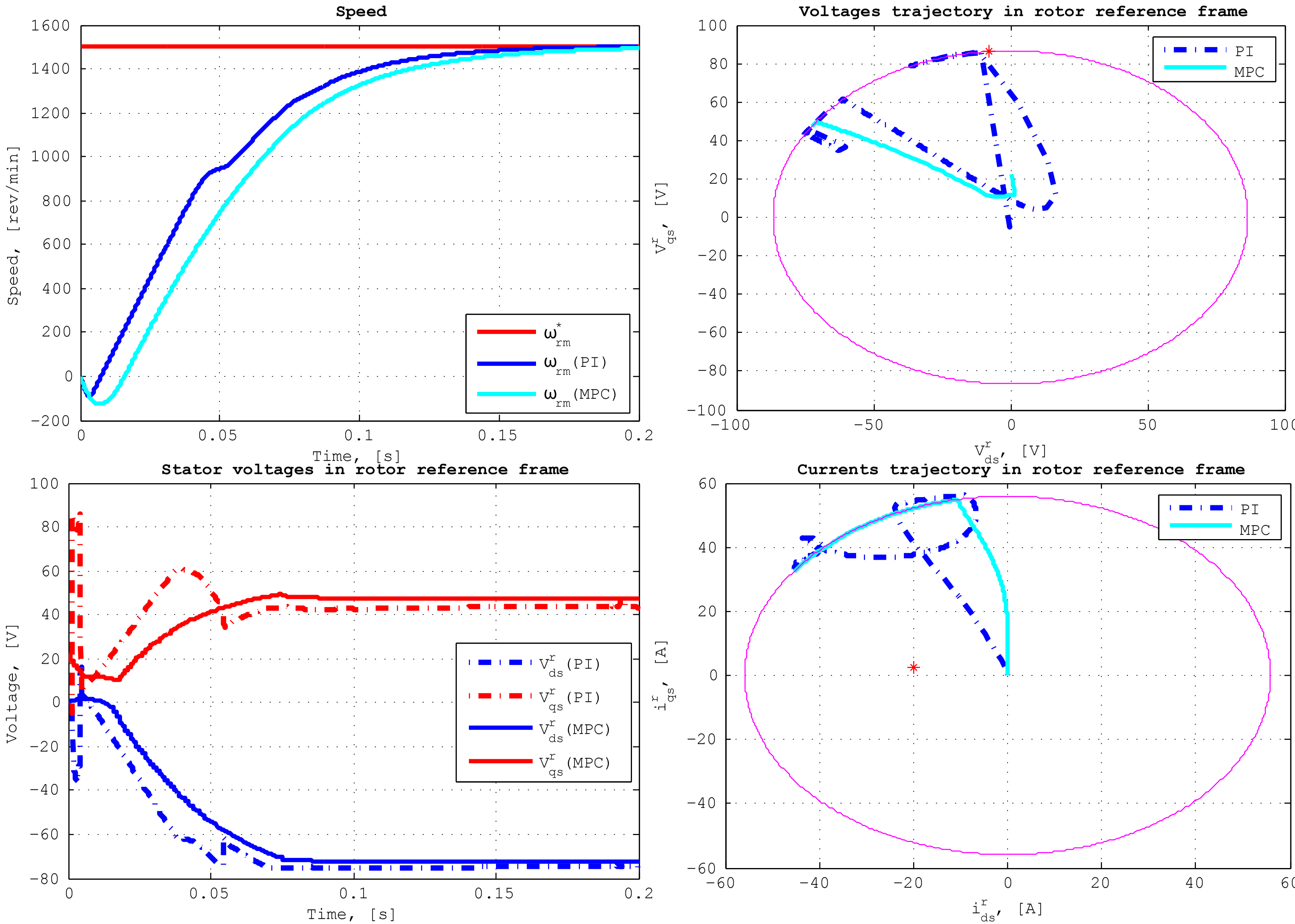

Fig. 4.5. Comparison test #2: $\omega^*_{rm} = 1500\,[rev/min]$, $\quad T_L = 0.6\,T_L^{max}\,[N \cdot m]$

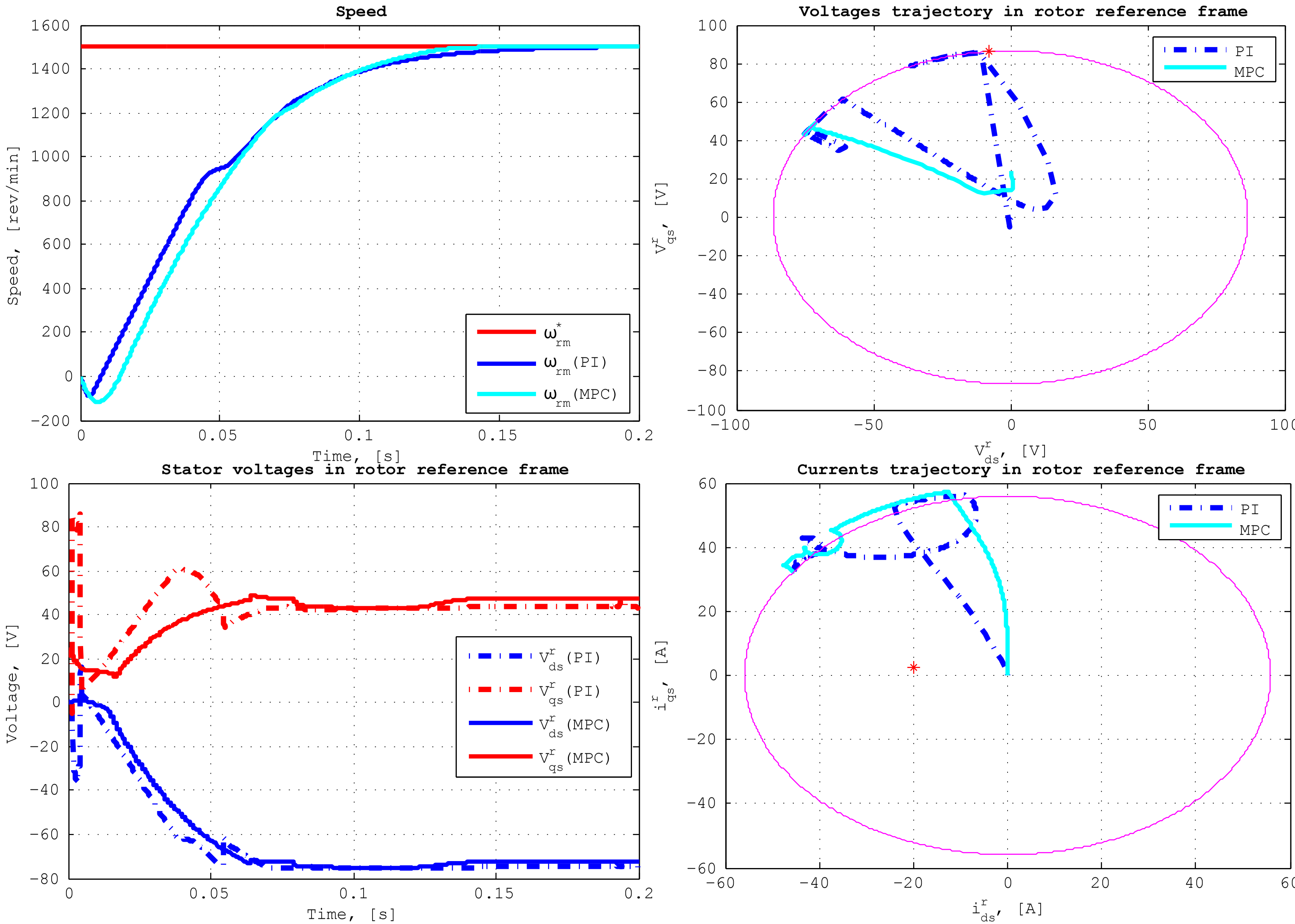

Fig. 4.6. Comparison test #3: $\omega^{*}_{rm} = 1500\,[rev/min]$, $\quad T_L = 0.6\,T_L^{max}\,[N \cdot m]$

CONCLUSIONS

1. The typical cascade structures are omitted.

2. The optimal control theory is applied to solve problem of speed control.

3. The speed controller synthesis is done on the base of nonlinear predictive model with respect to nonlinear voltage and current limitations.

4. The software system is built in order to perform real-time control optimization, using Matlab Optimization Toolbox.

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국문초록

본 논문은 모델 예측 제어 기법을 매입형 영구자석 동기 전동기에 적용하는 방법에 대하여 다루고 있다. 제안된 속도 제어기는 비선형적인 전압 및 전류 제한을 고려한 예측 모델에 기반하고 있으며, 속도와 전류를 포함한 모든 상태 공간 벡터를 다루는 다중 입출력(MIMO) 제어기의 형태로 구현되었다. 이론 및 실제적인 문제를 고려하여 제어기를 구성하였으며, 제어기의 구성 및 그에 따른 제어기 이득의 설정에 대한 내용은 본문에 자세히 설명되어 있다. 마지막으로 시뮬레이션을 수행하여 제안된 제어기의 성능을 비교 및 평가하였다.

주요어휘: 매입형 영구자석 동기 전동기, 속도 제어, 비선형 제어, 모델 예측 제어

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